Teko Classes, Maths : Suhag R. Kariya (S. R. K. Sir), Bhopal Phone : 0 903 903 7779, 0 98930 58881. page 1

# CAPACITANCE

## 1. INTRODUCTION

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

## 2. CAPACITANCE OF AN ISOLATED CONDUCTOR

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero). Potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor

$q \propto V$

$\Rightarrow$ $q = CV$

Where C is proportionality constant called capacitance of the conductor.

**2.1 Definition of capacitance :**

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

**2.2 Important points about the capacitance of an isolated conductor :**

**(i)** It is a scalar quantity.

**(ii)** Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

**(iii)** **1 Farad :** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, $1 nF = 10^{-9}$ F or $1\ pF = 10^{-12}$ F

**(iv)** **Capacitance of an isolated conductor depends on following factors :**

**(a) Shape and size of the conductor :**

On increasing the size, capacitance increases.

**(b) On surrounding medium :**

With increase in dielectric constant K, capacitance increases.

**(c) Presence of other conductors :**

When a neutral conductor is placed near a charged conductor capacitance of conductors increases.

**(v)** Capacitance of a conductor do not depend on

(a) Charge on the conductor

(b) Potential of the conductor

(c) Potential energy of the conductor.

## 2. POTENTIAL ENERGY OR SELF ENERGY OF AN ISOLATED CONDUCTOR

Work done in charging the conductor to the charge on it against its own electric field or total energy stored in electric field of conductor is called self energy or self potential energy of conductor.

**2.1 Electric potential energy (Self Energy) :**

$$U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

**2.2** Self energy is stored in the electric field of the conductor with energy density (Energy per unit volume)

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2 \text{ [The energy density in a medium is } \frac{1}{2}\varepsilon_0 \varepsilon_r E^2 \text{ ]}$$

where E is the electric field at that point.

**2.3** In case of charged conductor energy stored is only out side the conductor but in case of charged insulating material it is outside as well as inside the insulator.

**Ex.1** (i) When 10 coulomb charge is assigned to an isolated conductor its potential becomes 5 volt, find out capacitance of the conductor?

(ii) If now further 20 coulomb charge is supplied to it then what is the new potential on conductor?

**Sol.** (i) $C = \frac{Q}{V}$

$= \frac{10}{5} = 2$ Farad.

(ii) $V = \frac{Q}{C} = \frac{30}{2} = 15$ volt.

**Ex.2** An isolated conductor of 10 μF capacitance is given 10 μC charge. Find out stored energy and its potential?

**Sol.** Stored energy $U = \frac{1}{2}CV^2$

$$= \frac{1}{2}.\frac{Q^2}{C}. = \frac{1}{2}.\frac{(1\mu C)^2}{10\mu F}$$

$$= \frac{1\mu C}{20} = 0.05\ \mu J$$

Potential $V = \frac{Q}{C} = \frac{1\mu C}{10\mu F} = \frac{1}{10}$ volt.

**Q. 1** Potential of an isolated conductor is found to become 20 volt when a charge 20 μC is given to it, answer the following :

(i) What is the capacitance of conductor ?

(ii) If now further 40 μC charge is added to it then find out potential of conductor now?

(iii) What is the energy stored in two cases?

**Ans.** (i) 1 μF (ii) 60 V (iii) $U_i = 200\ \mu J$, $U_f = 1800\ \mu J$

## 3. CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

(i) If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\varepsilon_0 R$

R = Radius of spherical conductor. (may be solid or hollow.)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity.



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$C_{medium} = 4\pi\varepsilon_0 KR$

(iii) $\dfrac{C_{medium}}{C_{air\,/\,vaccum}} = K$ = dielectric constant.

**Ex. 3** 8 similar charged drops combine to from a bigger drop. The ratio of the capacity of bigger drop to that of smaller drop will be-

(A) 2 : 1 (B) 8 : 1 (C) 4 : 1 (D) 16 : 1

**Sol:** $C_{bigger\,drop} = (C_{small\,drop})n^{1/3}$ ....(1)

$n = 8$ ....(2)

$$\frac{C_{begger\ drop}}{C_{small\ drop}} = \frac{2}{1}$$

Hence the correct answer will be (A).

## 4. SHARING OF CHARGES ON JOINING TWO CHARGED CONDUCTORS

(i) Whenever there is potential difference there will be flow of charge.

(ii) Charge always have tendency to flows from **high potential energy** to **low potential energy** when released freely.

(iii) Positive charge always flows from **high potential** to **low potential** [if only electric force act on charge].

(iv) Negative charge always flows from **low potential** to **high potential** [if only electric force act on charge].

(v) The flow of charge will continue till there is potential difference between the conductors (finally potential difference = 0).

(vi) Formulae related with redistribution of charges.

| Before connecting the conductors | | |
|---|---|---|
| **Parameter** | **Conductor I[st]** | **Conductor II[nd]** |
| Capacitance | $C_1$ | $C_2$ |
| Charge | $Q_1$ | $Q_2$ |
| Potential | $V_1$ | $V_2$ |

| After connecting the conductors | | |
|---|---|---|
| **Parameter** | **I[st] Conductor** | **II[nd] Conductor** |
| Capacitance | $C_1$ | $C_2$ |
| Charge | $Q_1'$ | $Q_2'$ |
| Potential | V | V |

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \quad \Rightarrow \quad \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But, $Q_1' + Q_2' = Q_1 + Q_2$

$\therefore \quad V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$\therefore \quad Q_1' = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

& $\quad Q_2' = \dfrac{C_2}{C_1 + C_2}(Q_1 + Q_2)$

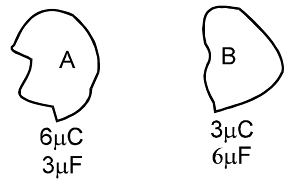
Heat loss during redistribution :

$$\Delta H = \frac{1}{2}\,\frac{C_1C_2}{C_1 + C_2}\,(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

**Note : Always put $Q_1$, $Q_2$, $V_1$ and $V_2$ with sign.**

**Ex. 4** A and B are two isolated conductors (that means they are placed at a large distance from each other). When they are joined by a conducting wire:




(i) Find out final charges on A and B ?

(ii) Find out heat produced during the process of flow of charges.

(iii) Find out common potential after joining the conductors by conducting wires?

**Sol.** (i) $Q_A' = \dfrac{3}{3+6}(6 + 3) = 3\mu C$

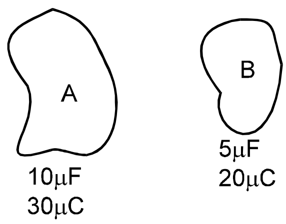
$Q_B' = \dfrac{6}{3+6}(6 + 3) = 6\mu C$

(ii) $\Delta h = \dfrac{1}{2}\cdot\dfrac{3\mu F.6\mu F}{(3\mu F + 6\mu F)}\cdot\left(2 - \dfrac{1}{2}\right)$

$= \dfrac{1}{2}.(2\mu F).\left(\dfrac{3}{2}\right) = \dfrac{3}{2}\mu J$

(iii) $V_C = \dfrac{3\mu C + 6\mu C}{3\mu F + 6\mu F} = 1\text{volt}.$

**Q. 2** When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.




(A) A to B (B) B to A (C) will not flow (D) can not say.

**Ans.** B.

**Q. 3** A conductor of capacitance 10μF connected to other conductor of capacitance 40 μF having equal charges 100 μC initially. Find out final voltage and heat loss during the process?

**Ans.** (i) V = 4V (ii) H = 225 μJ.

## 5. CAPACITOR :

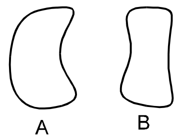
A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential decreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors




(a) $Q = CV$

$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

C = Capacitance of capacitor.

(b) Energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$$

This energy is stored inside the capacitor in its electric field with energy density

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 E^2 \text{ or } \frac{1}{2}\varepsilon_0 \varepsilon_r E^2 .$$

(v) The capacitor is represented as following:

(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor.

(b) Spherical capacitor.

(c) Cylindrical capacitor.

(vii) Capacitance of a capacitor depends

(a) Area of plates.

(b) Distance between the plates.

(c) Dielectric medium between the plates.

(viii) Capacitance of a parallel plate capacitor (air filled) is given by following formula

$$C = \frac{\varepsilon_0 A}{d}$$

where A = area of the plates.

d = distance between plates.

(ix) Electric field intensity between the plates of capacitors (air filled )

$E = \sigma/\varepsilon_0 = V/d$

(x) Force experienced by any plate of capacitor

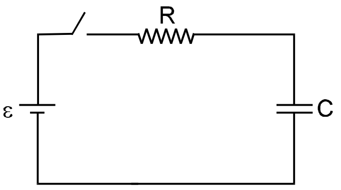
$F = q^2/2A\varepsilon_0$

## 6. CIRCUIT SOLUTION FOR R–C CIRCUIT AT t = 0 (INITIAL STATE)

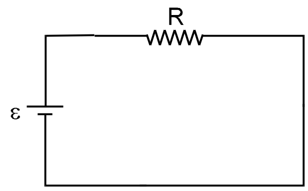
## AND AT t = ∞ (FINAL STATE)

**Note :** (i) Charge on the capacitor does not change instantaneously or suddenly if there if a resistance in the path (series) of the capacitor.

(ii) When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

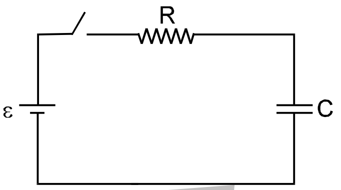


Before connection

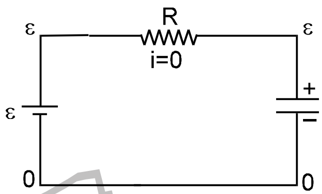


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor.



Before connection



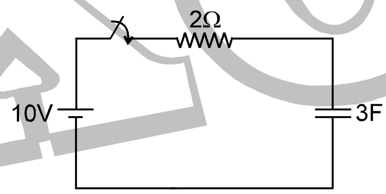
After connection at t = ∞

**Ex.4** A capacitor of capacity 1μF is charged to a potential difference of 1KV. The energy stored in the capacitor will be-

(A) 0.5 joule (B) 1 joule (C) 0.5 erg. (D) 1 erg.

**Sol.** $U = \frac{CV^2}{2} = \frac{10^{-6} \times 10^3 \times 10^3}{2} = 0.5$ Joule.

**Ex.5** Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

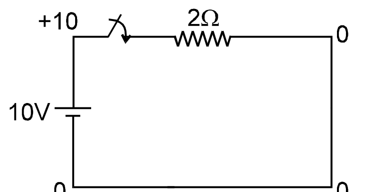



(a) Just after the switch is closed.

(b) After a long time when switch was closed.

**Sol.** **For just after closing the switch:**

potential difference across capacitor = 0



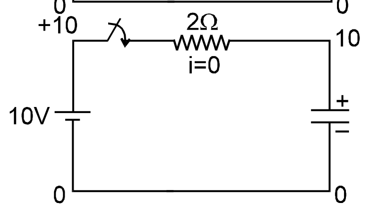

$\therefore Q_C = 0$ $\therefore i = \frac{10}{2} = 5A$

**After a long time**

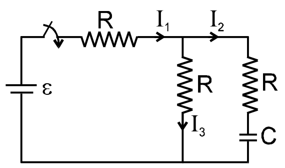
at steady state current i = 0

and potential difference across capacitor = 10 V

$\therefore Q_C = 3 \times 10 = 30$ C




**Ex.6** Find out current $i_1$, $i_2$, $i_3$, charge on capacitor and $\frac{dq}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.




(a) Just after the switch is closed

(b) After a long time when switch is closed.



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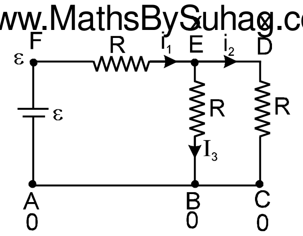
**Sol.** Initially the capacitor is uncharged so its behaviour is like a conductor




Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's I[st] law at point E :

$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0$$

$$\frac{3x}{R} = \frac{\varepsilon}{R}$$

$$x = \frac{\varepsilon}{3} \qquad Q_c = 0$$

$$\therefore \quad I_1 = \frac{-\varepsilon/3+\varepsilon}{R} = \frac{2\varepsilon}{3R}$$

$$I_2 = \frac{\varepsilon}{3R}$$

$$I_3 = \frac{\varepsilon}{3R}$$

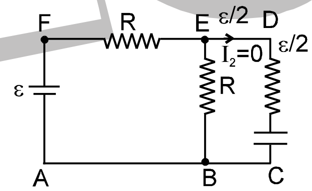
**Alternatively**

$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R+\frac{R}{2}} = \frac{2\varepsilon}{3R}$$

$$i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R}$$

**at t = ∞ (finally)**

capacitor completely charged so their will be no current through it.



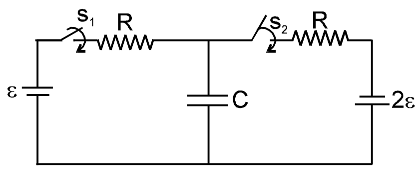

$$I_2 = 0, \quad I_1 = I_3 = \frac{\varepsilon}{2R}$$

$$V_e - V_B = V_D - V_C = (\varepsilon/2R)R = \varepsilon/2$$

$$\Rightarrow \quad Q_C = \frac{\varepsilon C}{2}, \qquad \frac{dQ}{dt} = I_2 = 0$$

| Time | $I_1$ | $I_2$ | $I_3$ | Q | dQ /dt |
|---|---|---|---|---|---|
| **Time t = 0** | $\frac{2\varepsilon}{3R}$ | $\frac{\varepsilon}{3R}$ | $\frac{\varepsilon}{3R}$ | 0 | $\frac{\varepsilon}{3R}$ |
| **Finally t = ∞** | $\frac{\varepsilon}{2R}$ | 0 | $\frac{\varepsilon}{2R}$ | $\frac{\varepsilon C}{2}$ | 0 |

**Ex.7** At t = 0 switch $S_1$ is closed and remains closed for a long time and $S_2$ remains open. Now $S_1$ is opened and $S_2$ is closed assuming that capacitor is initially uncharged find out

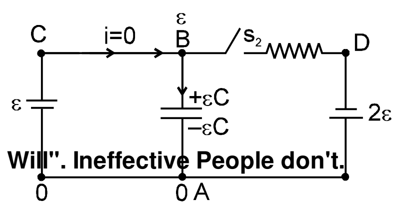



(i) The current through the capacitor immediately after that moment

(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after $S_2$ is closed.

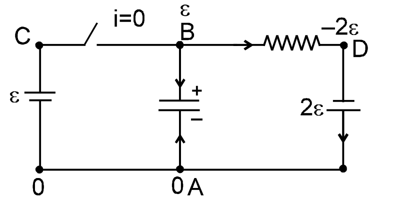
**Sol.** (i) Let Potential at point A is zero. Then at point B and C it will be ε (because current through the circuit is zero).




$$V_B - V_A = C(\varepsilon - 0)$$

$\therefore$ Charge on capacitor = $C(\varepsilon - 0) = C\varepsilon$

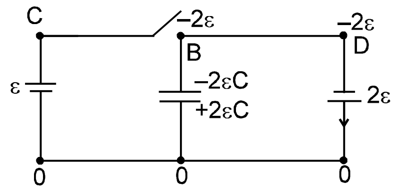
(ii) Now $S_2$ is closed and $S_1$ is open. (p.d. across capacitor and charge on it will not change suddenly)




Potential at A is zero so at D it is $-2\varepsilon$.

$\therefore$ current through the capacitor = $\dfrac{\varepsilon - (-2\varepsilon)}{R} = \dfrac{3\varepsilon}{R}$ (B to D)

(iii) after a long time i = 0




$V_B - V_A = V_D - V_A = -2\varepsilon$

$\therefore$ $Q = C(-2\varepsilon - 0) = -2\varepsilon C$

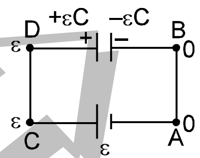
The charge on the lower plate (which is connected to the battery) changes from $-\varepsilon C$ to $2\varepsilon C$.

$\therefore$ this charge will come form the battery,

$\therefore$ charge flown from that cell is $3\varepsilon C$ downward.

**Ex.8** A capacitor of capacitance C which is initially uncharged is connected with a battery. Find out heat dissipated in the circuit during the process of charging.

**Sol.** Final status




Let potential at point A is 0, so at B also 0 and at C and D it is ε. finally, charge on the capacitor

$Q_C = \varepsilon C$

$U_i = O$

$U_f = \dfrac{1}{2} CV^2 = \dfrac{1}{2} C\varepsilon^2$

work done by battery $= \int p dt$

$w = \int \varepsilon i dt$

$= \varepsilon \int i dt$

$= \varepsilon \,.\, Q$

$= \varepsilon.\ \varepsilon C$

$= \varepsilon^2 C$

(Now onwards remember that w.d. by battery = εQ if Q has flown out of the cell from high potential and w.d. on battery is εQ if Q has flown into the cell through high potential)

Heat produced = $W - (U_f - U_i) = \varepsilon^2 C - \dfrac{1}{2}\varepsilon^2 C = \dfrac{C\varepsilon^2}{2}$.

**Ex.9** A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε such that the positive terminal of battery is connected with positive plate of capacitor. Find out heat loss in the circuit during the process of charging.

**Sol.**



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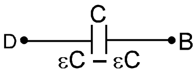
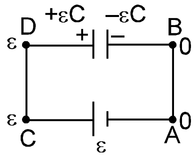



Since the initial and final charge on the capacitor is same before and after connection.

Here no charge will flow in the circuit so heat loss = 0

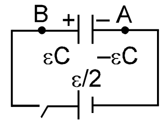
**Ex.10** A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε/2 such that the positive terminal of battery is connected with positive plate of capacitor. After a long time

(i) Find out total charge flow through the battery

(ii) Find out total work done by battery

(iii) Find out heat loss in the circuit during the process of charging.

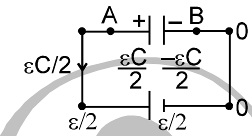
**Sol.** Let potential of A is 0 so at B it is $\frac{\varepsilon}{2}$. So final charge on capacitor = Cε/2




Charge flow through the capacitor = (Cε/2 – Cε) = –Cε/2

So charge is entering into battery.

finally,




Change in energy of capacitor = $U_{final} - U_{initial}$

$$= \frac{1}{2}C\left(\frac{\varepsilon}{2}\right)^2 - \frac{\varepsilon^2 C}{2}$$

$$= \frac{1}{8}\varepsilon^2 C - \frac{1}{2}\varepsilon^2 C = -\frac{3\varepsilon^2 C}{8}$$

Work done by battery $= \frac{\varepsilon}{2} \times \left(-\frac{\varepsilon C}{2}\right)$

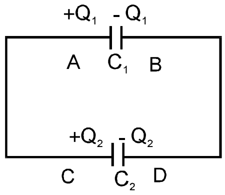
$$= -\frac{\varepsilon^2 C}{4}$$

Work done by battery = Change in energy of capacitor + Heat produced

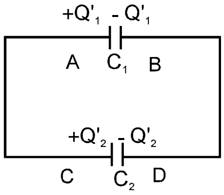
Heat produced $= \frac{3\varepsilon^2 C}{8} - \frac{\varepsilon^2 C}{4} = \frac{\varepsilon^2 C}{8}$

## 6. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are $C_1$ and $C_2$ are connected as shown in figure



Initially



Finally

**Before connecting the capacitors**

| Parameter | Capacitor I[st] | Capacitor II[nd] |
|---|---|---|
| Capacitance | $C_1$ | $C_2$ |
| Charge | $Q_1$ | $Q_2$ |
| Potential | $V_1$ | $V_2$ |

**After connecting the capacitors**

| Parameter | I[st] Capacitor | II[nd] Capacitor |
|---|---|---|
| Capacitance | $C_1$ | $C_2$ |
| Charge | $Q'_1$ | $Q'_2$ |
| Potential | V | V |

(a) Common potential :

By charge conservation of plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$\Rightarrow \quad V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{\text{Total charge}}{\text{Total capacitance}}$

(b) $Q_1' = C_1V = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

$Q_2' = C_2V = \dfrac{C_2}{C_1 + C_2}(Q_1 + Q_2)$

(c) Heat loss during redistribution :

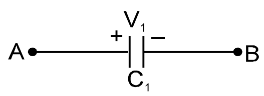
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

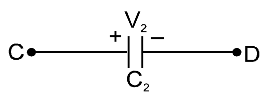
The loss of energy is in the form of Joule heating in the wire.

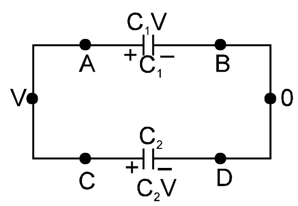
**Note :** (i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values ($Q_1$, $Q_2$, $V_1$, $V_2$) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

**Derivation of above formulae :**




Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$C_1V + C_2V = C_1V_1 + C_2V_2$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

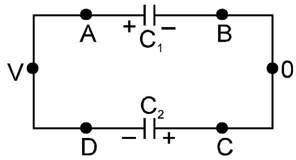
$$H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$




when oppositely charge terminals are connected then

$\therefore \quad C_1V_1 + C_2V_2 = C_1V_1 - C_2V_2$

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

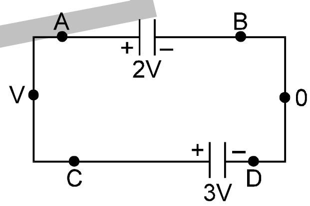
$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$$

**Ex.11** Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.


**Sol.**




Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$3V + 2V = 40 + 30$

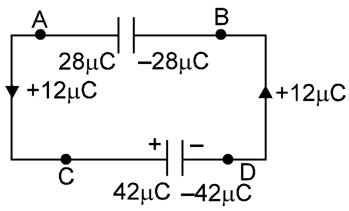
$5V = 70$

$V = 14$ volt

Charge flow

$= 40 - 28$

$= 12\ \mu C$




Now final charges on each plate

(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

$= 400 + 150 - 490$

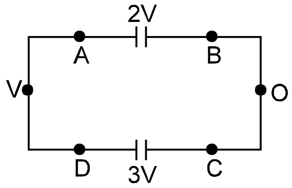
$= 550 - 490$

$= 60$ J

**Note 1.** When capacitor plates are joined then the charge remains conserved.

**Note 2.** We can also use direct formula of redistribution as given above.

**Ex.11** Repeat above question if A is connected with D and B is connected with C.

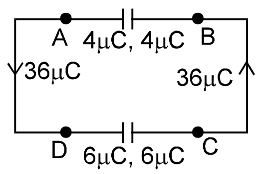



**Sol.** Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + 3V = 10$

$\Rightarrow \quad V = 2$

Now charge on each plate




Heat produced $= 400 + 150 - \frac{1}{2} \times 5 \times 4^2$

$= 550 - 10$

$= 540$ J

**Note :** here heat produced is more. Think why?

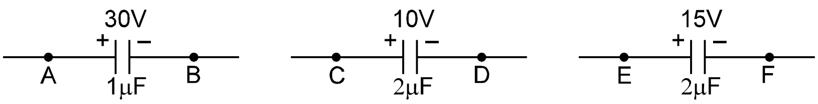
**Ex.13** A 20μF capacitor is charged to potential of 500V and then connected in parallel to another capacitor of capacity 10μF. If the potential of 10μF capacitor is 200 Volt then the common potential of two will be -

(1) 100 V (B) 200 V (C) 300 V (D) 400 V

**Sol.** $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

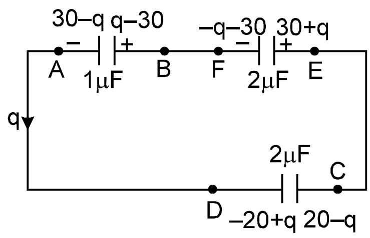
$= \dfrac{20\times10^{-6}\times500 + 10\times10^{-6}\times200}{20\times10^{-6} + 10\times10^{-6}} = 400$ V

**Ex.14** Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged upto potential difference 20 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.




**Sol.** Let charge flow is q.

Now applying kirchhoff's voltage low

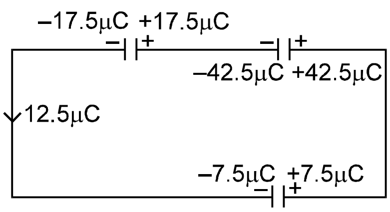



$-\dfrac{(q-20)}{2} - \dfrac{(30+q)}{2} - \dfrac{30-q}{1} = 0$

$-2q = -25$

$q = 12.5\ \mu C$

Final charges on plates

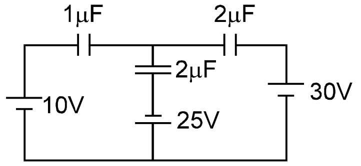




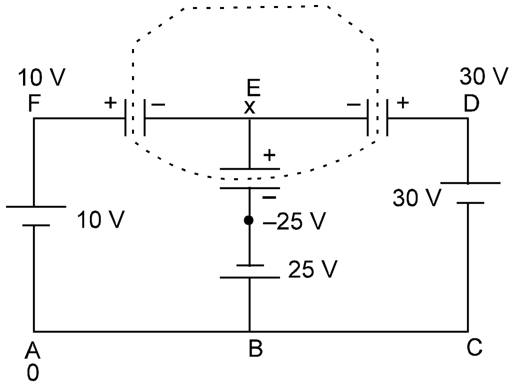


**Ex.15** In the given circuit find out the charge on each capacitor. (Initially they are uncharged)




**Sol.**




Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V. Now apply kirchhoff's I[st] law at point E. ( total charge of all the plates connected to 'E' must be same as before i.e. 0)

$\therefore \quad (x - 10) + (x - 30)2 + (x + 25)2 = 0$

$5x = 20$

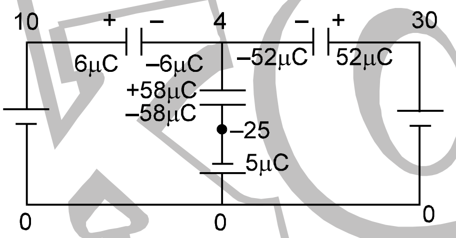
$x = 4$

Final charges :

$Q_{2\mu F} = (30 - 4)2 = 52\ \mu C$

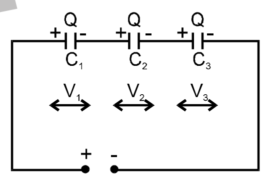
$Q_{1\mu F} = (10 - 4) = 6\mu C$

$Q_{2\mu F} = (4 - (-25))2 = 58\ \mu C$




## 7. COMBINATION OF CAPACITORS :

### 7.1 Series Combination :




(i) When initially uncharged capacitors are connected as shown in the combination is called series combination.

(ii) All capacitors will have same charge but different potential difference across then.

(iii) We can say that

$$V_1 = \frac{Q}{C_1}$$

$V_1$ = potential across $C_1$

Q = charge on positive plate of $C_1$

$C_1$ = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2}\ ,\ V_3 = \frac{Q}{C_3}\ ........$$

(iv) $V_1 : V_2 : V_3 = \dfrac{1}{C_1} : \dfrac{1}{C_2} : \dfrac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$$V \propto \frac{1}{C}$$

**Note :** In series combination the smallest capacitor gets maximum potential.

(v)
$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance :

Equivalent capacitance of any combination is that capacitance which when connected in place of the combination stores same charge and energy that of the combination.

In series :

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \; .......$$

**Note :** In series combination equivalent is always less the smallest capacitor of combination.

(vii) Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

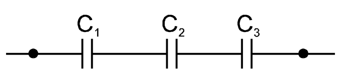
Energy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \,.\, \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

**Note :** Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

**Derivation of Formulae :**



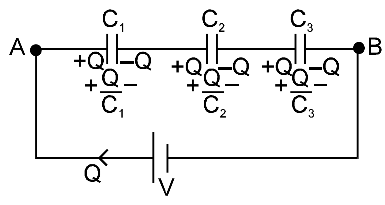

meaning of equivalent capacitor


$$C_{eq} = \frac{Q}{V}$$

Now,

Initially, the capacitor has no charge.

Applying kirchhoff's voltage law




$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

in general

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

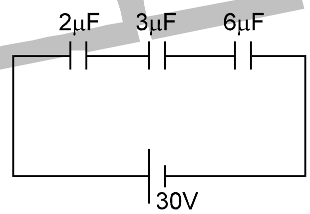
**Ex.16** Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following:-

(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) $U_{total}$ in capacitors

(iv) heat produced in the circuit




**Sol.** $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$

$C_{eq} = 1\mu F$.

(i) $Q = C_{eq} V = 30\mu C$.

(ii) charge on 3μF capacitor = 30μC

energy = $\frac{Q^2}{2C} = \frac{30 \times 30}{2 \times 3} = 150\mu J$

(iii) $U_{total} = \frac{30 \times 30}{2}\ \mu J$

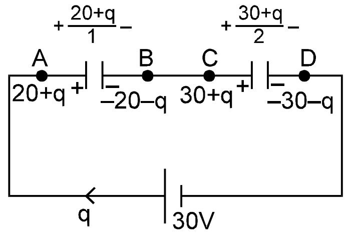
$= 450\ \mu J$

(iv) Heat produced = (30 μC) (30) – 450 μJ

= 450 μJ.

**Ex.17** Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor


**Sol.**




Now applying kirchoff voltage law

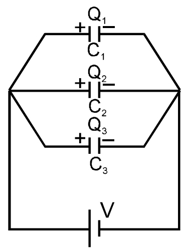
$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$-40 - 2q - 30 - q = -60$

$3q = -10$

Charge flow = $-10/3\ \mu C$.

**7.2 Parallel Combination :**




(i) When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.

(ii) All capacitors have same potential difference but different charges.

(iii) We can say that :

$Q_1 = C_1V$

$Q_1$ = Charge on capacitor $C_1$

$C_1$ = Capacitance of capacitor $C_1$

V = Potential of across capacitor $C_1$

(iv) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance

$Q \propto C$

(v) $Q_1 = \dfrac{C_1}{C_1 + C_2 + C_3} Q$

$Q_2 = \dfrac{C_2}{C_1 + C_2 + C_3} Q$

$Q_3 = \dfrac{C_3}{C_1 + C_2 + C_3} Q$

Where $Q = Q_1 + Q_2 + Q_3$ ......

**Note :** Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination

$C_{eq} = C_1 + C_2 + C_3$

**Note :** Equivalent capacitance is always greater than the largest capacitor of combination.

(vii) Energy stored in the combination :

$$V_{combination} = \frac{1}{2} C_1V^2 + \frac{1}{2} C_2V^2 + .... = \frac{1}{2} (C_1 + C_2 + C_3 .....) V^2$$

$$= \frac{1}{2} C_{eq}V^2$$

$U_{battery} = QV = CV^2$

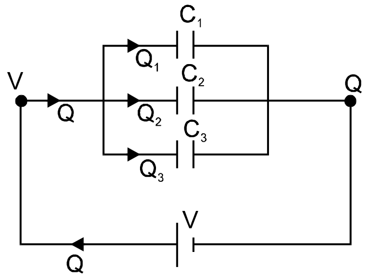


$$= \frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

**Note :** Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

**Formulae Derivation for parallel combination :**




$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= V(C_1 + C_2 + C_3)$$

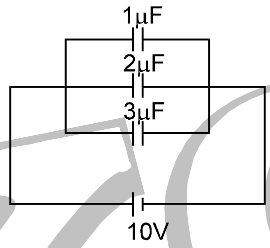
$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

In general

$$C_{eq} = \sum_{n=1}^{n} C_n$$

**Ex.18** Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following




(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.

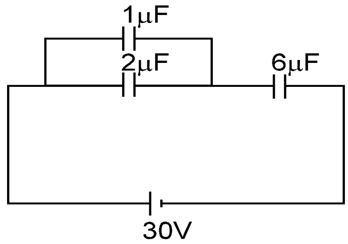
**Sol.** (i) $Q = (30 + 20 + 10)\mu C$
$= 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

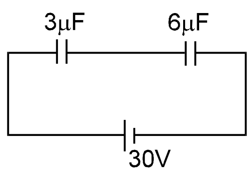
(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ J$

**Ex.19** In the given circuit find out charge on 6μF and 1μF capacitor.




**So.** It can be simplified as




$C_{eq} = \frac{18}{9} = 2\mu F$

charge flow through the cell $= 30 \times 2\ \mu C$

$Q = 60\ \mu C$

Now charge on 3μF = Charge on 6μF= 60 μC

Potential difference across 3μF

$= 60/ 3= 20\ \ V$

$\therefore$ Charge on 1μF = 20 μC.



### 7.3 Mixed Combination :

The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category.

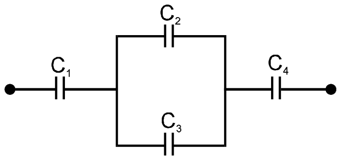
There are two types of mixed combinations

(i) Simple (ii) Complex.

### 7.4 Simple Mixed Combination :

Combinations which can be easily converted in series parallel combination.

**Ex.20**




The given combination is neither a series nor a parallel combination but $C_2$ and $C_3$ are in parallel and that is in series with $C_1$ and $C_4$.

**Ex.21** Two condensers of same capacity are first connected in parallel and then in series. The ratio of resultant capacities in two cases will be-

(A) 1 : 4 (B) 4 : 1 (C) 1 : 2` (D) 2 : 1

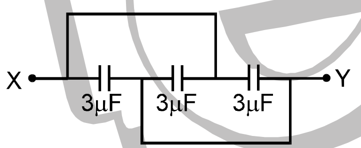
**Sol.** $C_P = C + C = 2C$

$$C_S = \frac{CC}{C+C} = \frac{C}{2}$$

$$\frac{C_P}{C_S} = \frac{2C}{C/2} = \frac{4}{1}$$

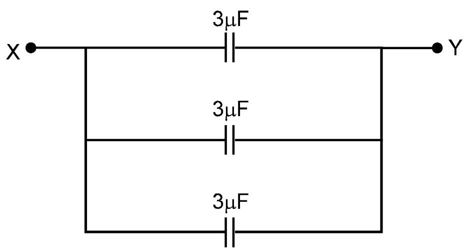
Hence the correct answer will be (B).

**Ex.22** The equivalent capacity in the adjoining figure between the point X and Y will be-




(A) 4.5 μF (B) 9μF (C) 1 μF (D) 6μF

**Sol.** Equivalent circuit ( ∴ all capacitors are connected between same p.d. x and y )




All the three condensers are connected in parallel, hence the resultant capacity

= 3 + 3 + 3

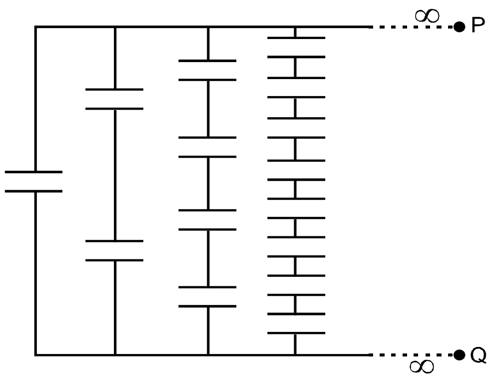
= 9μF

Hence the correct answer will be (B).

**Ex.23** In the adjoining diagram if the capacity of each condenser is 1μF then the resultant capacity between the points P and Q will be -






(A) ∞ (B) zero (C) 2μF (D) 8μF

**Sol.** The different branches of the condensers have number of condensers in G.P., hence the equivalent capacity

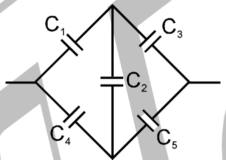
$$C' = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$$

$$C' = \frac{1}{1-\frac{1}{2}} = 2\mu F.$$

Hence the correct answer will be (C).

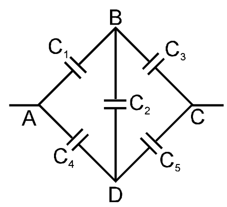
### 7.5 Complex Mixed Combination :

**Ex. 24**




The given combination can not be simplified by series or parallel combination such a combination is solved by using kirchhoff's law and other techniques. A special case of this combination is Wheatstone bridge.

### 7.6 Wheat store bridge :



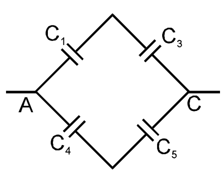

If $\frac{C_1}{C_4} = \frac{C_3}{C_5}$ or $C_1C_5 = C_3C_4$ then

(i) Such combination is called balanced Wheatstone bridge.

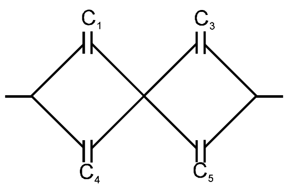
(ii) In this case $V_B = V_D$ .

(iii) Change on $C_2 = 0$

(iv) The equivalent can be converted in to given circuits:




or




(v) $C_{eq} = \frac{(C_1 + C_4)(C_3 + C_5)}{C_1 + C_2 + C_3 + C_5}$

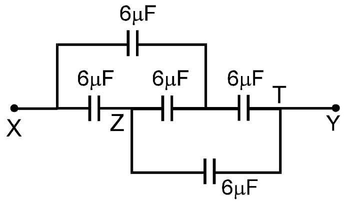
 

$$= \frac{C_1C_3}{C_1 + C_3} + \frac{C_4C_5}{C_4 + C_5}$$

(vi) If $C_1 = C_4 = C_3 = C_5 = C$

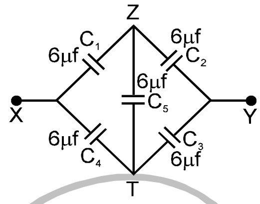
then $C_{eq} = C$

**Ex.25** The equivalent capacity between the points X and Y in the following circuit (Figure) will be -




(A) 6 mF (B) 1 μF (C) 24 μF (D) 3 μμF

**Sol.** Because the bridge is balanced, hence the central capacitance between Z and T is ineffective. $C_1$ and $C_2$ are connected in series, hence their resultant $C' = \frac{C}{2} = 3\mu F$ similarly $C_3$ and $C_4$ are connected in series, hence their resultant $C'' = \frac{C}{2} = 3\mu F$. Now the two branches are connected in parallel




$\therefore$ $C_{eq.} = 3 + 3 = 6\mu F$

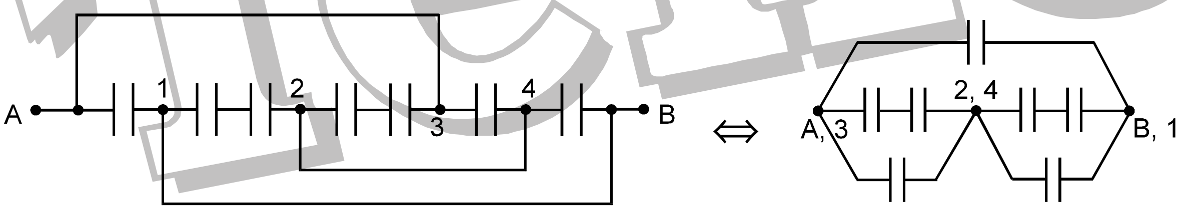
Hence the correct answer will be (A).

## 7.7 Other important circuit solving techniques :

(Applicable in both capacitive and resistive networks).

**(a) Equipotential Technique**

All the junctions which are at equal potential (such as junctions connected by a connecting wire) can be replaced by a single junction. So redraw the circuit to get it simplified

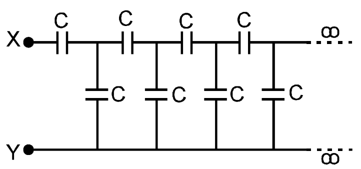



**(b) Infinite Circuits**

Assume equivalent capacitance/resistance to be $C_{eq}/R_{eq}$ of whole network, then add one more branch (repetitive) in infinite network. Calculate equivalent of this new circuit. It should again be equal to $C_{eq}/R_{eq}$.

**Note:-** If all the resistance/capacitances of a circuit are made K times then equivalent will also become K-times.

**Ex.26** In the adjoining figure, the effective capacity of the group of condensers will be-




(A) nC (B) ∞ (C) zero (D) 0.62 C

**Sol.** As the combination is spreading upto infinity, hence the capacity will remain same at last but one step also. Let the capacity of the combination is C´.

$\therefore \quad \frac{1}{C'} = \frac{1}{C} + \frac{1}{C + C'}$

$\frac{1}{C'} = \frac{C'+2C}{C[C'+C]}$

or $\quad C'^2 + 2CC' - CC' - C' = 0$

or $\quad C'^2 + CC' - C^2 = 0$

This is a quadratic equation in C´

$\therefore \quad C' = \frac{-C \pm C\sqrt{5}}{2}$

Negative capacity is impossible

$\therefore \quad C' = 0.62\ C$

Hence the correct answer will be (D).

**(d) Kirchhoff's Laws**

(i) **J**unction **R**ule

Sum of charges present on the plates of capacitors connected at a junction is equal to zero (If initially all the capacitors are uncharged) (while adding charges of different plates battery can be neglected as net charge on battery is zero).

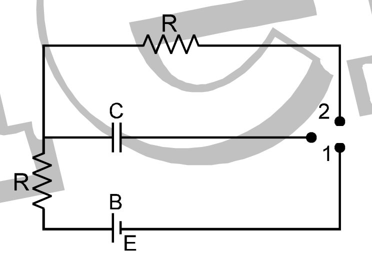
(ii) **L**oop **R**ule

In any closed loop the algebraic sum of potential drops across different elements is equal to zero.

## 8. CHARGING AND DISCHARGING OF A CAPACITOR

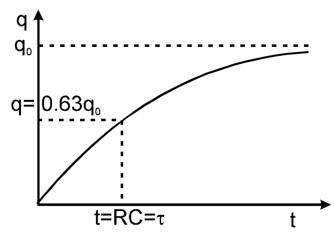
### 8.1 Charging of a condenser :

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$




Where $q_0$ = maximum final value of charge at $t = \infty$.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.




(ii) If $t = RC = \tau_1$ then

$q = q_0\ [1 - e^{-(RC/RC)}] = q_0 \left[1 - \frac{1}{e}\right]$

or $\quad q = q_0\ (1 - 0.37) = 0.63\ q_0$

$= 63\%$ of $q_0$

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

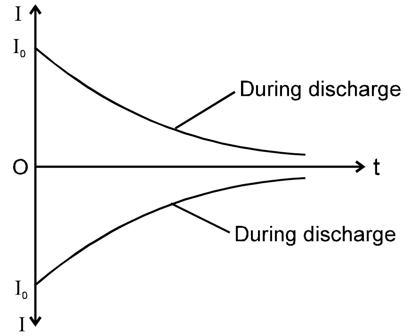
According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If $t = RC = \tau$ = Time constant

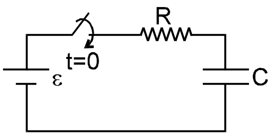
$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\ I_0$

= 37% of $I_0$

i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.



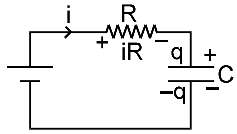

**Derivation of formulae for charging of capacitor**




it is given that initially capacitor is uncharged.

let at any time

Applying kirchoff voltage law




$$\varepsilon - iR - \frac{q}{C} = 0$$

$$iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{CR}{\varepsilon C - q} \cdot dq = dt.$$

$$\int_O^q \frac{dq}{\varepsilon C - q} = \int_O^t \frac{dt}{RC}$$

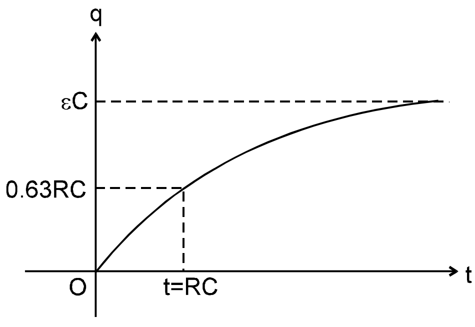
$$-\ln(\varepsilon C - q) + \ln C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = e^{t/RC}$$

$\varepsilon C - q = \varepsilon C \cdot e^{-t/RC}$

$q = \varepsilon C(1 - e^{-t/RC})$

RC = time constant of the RC series circuit.

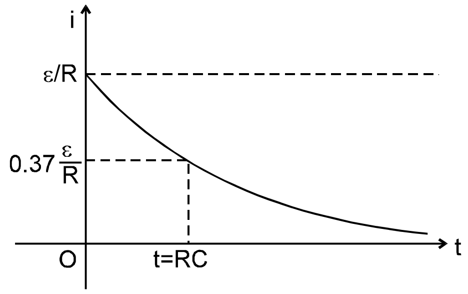


**After one time constant**

$$q = \varepsilon C\left(1-\frac{1}{e}\right)$$

$$\varepsilon C\ (1 - 0.37) = 0.63\ \varepsilon C.$$

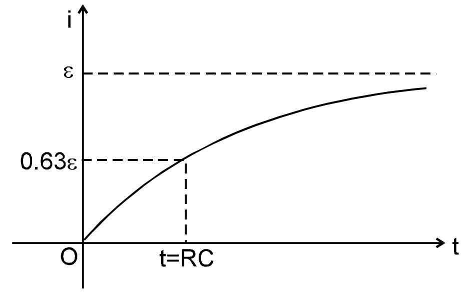
**Current at any time t**




$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

$$= \frac{\varepsilon}{R}e^{-t/RC}$$

**Voltage across capacitor after one time constant V = 0.63 ε**




$Q = CV$

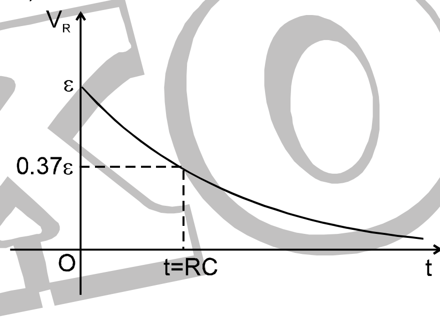
$V_C = e(1 - e^{-t/RC})$

**Voltage across the resistor**

$V_R = iR$

$= \varepsilon e^{-t/RC}$

Heat (by energy conservation = work done by battery – ΔUcapacitor )




$$= c\varepsilon(\varepsilon) - \left(\frac{1}{2}\ (\varepsilon^2 - 0)\right)$$

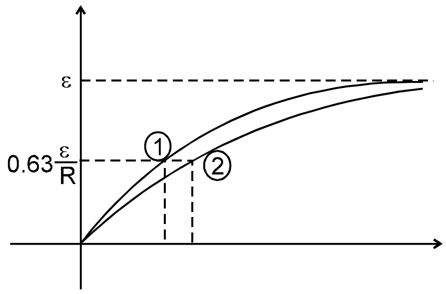
$$= \frac{1}{2}c\varepsilon^2$$

**Alternatively :**

$$\text{Heat} = H = \int_0^\infty i^2 R\,dt$$

$$= \int_0^\infty \frac{\varepsilon^2}{R^2}\ e^{-\frac{2t}{RC}}\ R\ dt$$

$$= \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$




$$= \frac{\varepsilon^2}{R}\ \frac{e^{-\frac{2t}{RC}}}{-2/RC}\Bigg|_0^\infty$$

$$= \frac{\varepsilon^2 RC}{2R}\ e^{-\frac{2t}{RC}}\Bigg|_0^\infty$$

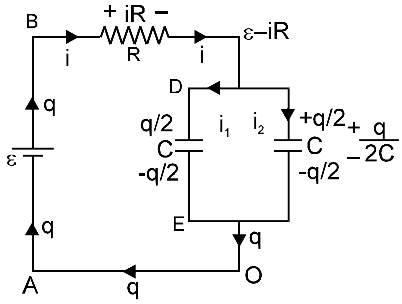
$$= \frac{\varepsilon^2 C}{2}$$

In the figure time constant of (2) is more than (1).



**Ex.27** Without using the formula of equivalent. Find out charge on capacitor and current in all the branches as a function of time.




Applying KVL in ABDEA

$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR}$$

$$= \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$-\int_0^q (2\varepsilon C - q)\, dq = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

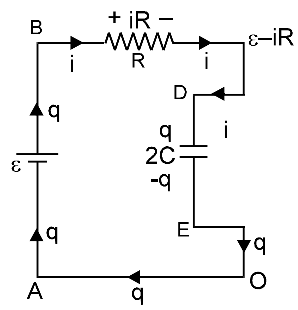
$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\, e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\, e^{-t/RC}$$

**Alternate solution**

by equivalent




Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below

$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\, e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\, e^{-t/RC}$$



**Ex.28** A capacitor is connected to a 12 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1μs. Find the capacitances of the capacitor.

**Sol.** The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at t = 1 μs, the potential difference is 4V whereas the steady potential difference is $Q_0/C$ = 12V. So,

$$4V = 12V(1 - e^{-t/RC})$$

or, $$1 - e^{-t/RC} = \frac{1}{3}$$

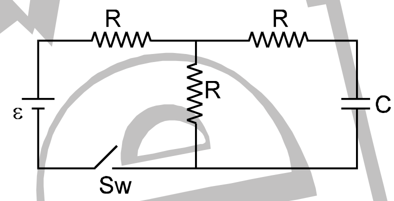
or, $$e^{-t/RC} = \frac{2}{3}$$

or, $$\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405$$

or, $$RC = \frac{t}{0.405} = \frac{1\mu s}{0.45} = 2.469\ \mu s$$

or, $$C = \frac{2.469\mu s}{10\Omega} = 0.25\ \mu F.$$

**Ex.29** Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.

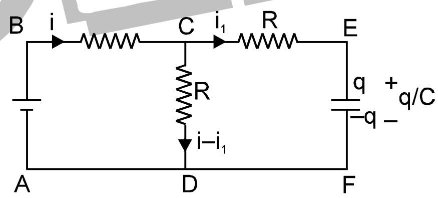



**Sol.** Applying KVL in loop ABCDA

$$\varepsilon - iR - (i - i_1)R = 0$$

$$\varepsilon - 2iR + i_1R = 0$$

Applying KVL in loop ABCEFDA




$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

$$\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C}$$

$$\varepsilon C - 3i_1RC = 2q$$

$$\varepsilon C - 2q = 3\frac{dq}{dt}.RC$$

$$\int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$$

$$-\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC}$$

$$q = \frac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$$

**Method for objective :**

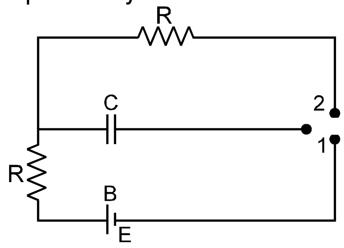
In any circuit when there is only one capacitor then

$q = Q_{st}\left(1 - e^{-t/\tau}\right)$ ; $Q_{st}$ = steady state charge on capacitor (has been found in article 6 in this sheet)

$\tau = R_{eff}.\ C$

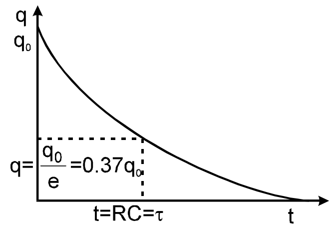
Reffective is the resistance between the capacitor when battery is replaced by its internal resistance.

**8.2 Discharging of a condenser :**




(i) In the above circuit (in article 8.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\ e^{-(t/RC)}$
i.e. the charge falls exponentially.




(iii) If t = RC = τ = time constant, then

$$q = \frac{q_0}{e} = 0.37q_0 = 37\% \text{ of } q_0$$

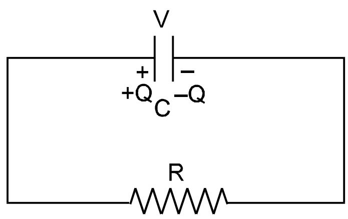
i.e. the time constant is that time during which the charge on condenser plates discharge process falls to 37%

(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

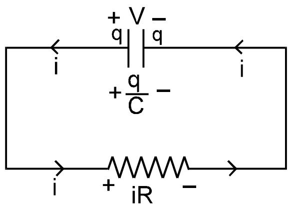
(v) The potential difference across the condenser plates at any instant of time t is given by
$V = V_0e^{-(t/RC)}$ Volt.

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.
i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current.

**Derivation of equation of discharging circuit :**



Initially



At any time t

Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

$$i = \frac{q}{CR}$$

$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$



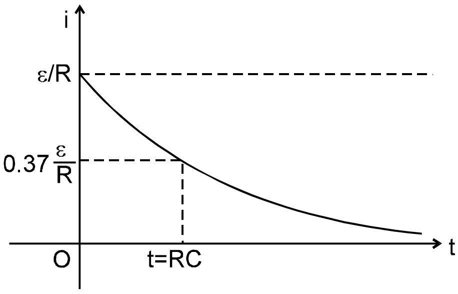


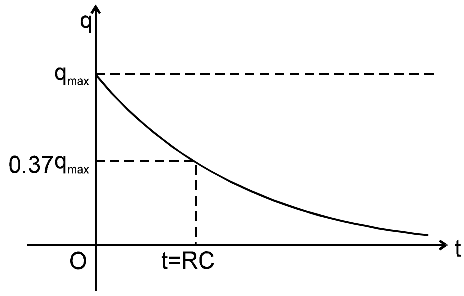
 

$\Rightarrow \quad -\ln \frac{q}{Q} = +\frac{t}{RC}$

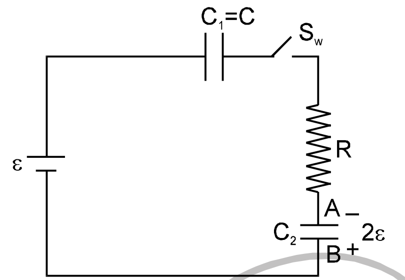
$q = Q\,.\,e^{-t/RC}$

$i = -\frac{dq}{dt} = \frac{Q}{RC} e^{-t/RC}$

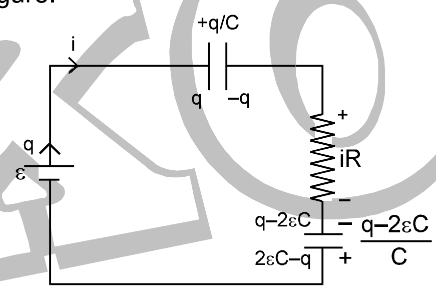



**Ex.30** At t = 0 $S_w$ is closed, if initially $C_1$ is uncharged and $C_2$ is charged to a potential difference 2ε then find out following
(Given $C_1 = C_2 = C$)
(a) Charge on $C_1$ and $C_2$ as a function of time.
(b) Find out current in the circuit as a function of time.
(c) Also plot the graphs for the relations derived in part (b).




**Sol.** Let q charge flow in time 't' from the battery as shown.
The charge on various plates of the capacitor is as shown in the figure.
Now applying KVL




$\varepsilon - \frac{q}{C} - iR - \frac{q-2\varepsilon C}{C} = 0$

$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$

$3\varepsilon = \frac{2q}{C} + iR$

$3\varepsilon - iR = \frac{2q}{C}$

$3\varepsilon - iRC = 2q$

$\frac{dq}{dt} RC = 3\varepsilon C - 2q$

$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC}$

$-\frac{\ln(3\varepsilon C - 2q)}{2} = \frac{t}{RC}$

$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC}$

$3\varepsilon C - 2q = 3\varepsilon C\ e^{-2t/RC}$

$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q$

$q = \frac{3}{2}\varepsilon C\ (1 - e^{-2t/RC})$ **Ans.**

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}\ e^{-2t/RC}$$ **Ans.**

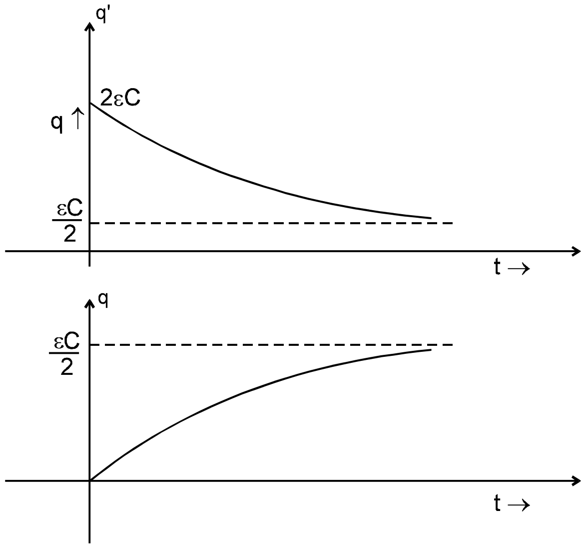
on the plate B

$$q' = 2\varepsilon C - q$$

$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C\ e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2} + \frac{3}{2}\ \varepsilon C e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$$




**Ex.31** The electric field between the plates of a parallel–plate capacitance 2.0 μF drops to one third of its initial value in 4.4 μs when the plates are connected by a thin wire. Find the resistance of the wire.

**Sol.** The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_0} = \frac{Q_0}{A\varepsilon_0}\ e^{-t/RC}$$

or, $E = E_0 e^{-t/RC}$

In the given problem, $E = \frac{1}{3}E_0$ at t = 4.4 μs.

Thus, $\frac{1}{3} = e^{-\frac{4.4\mu s}{RC}}$

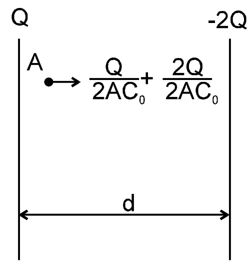
or, $\frac{4.4\mu s}{RC} = \ln 3 = 1.1$

or, $R = \frac{4.4\mu s}{1.1 \times 2.0\mu F} = 2.0\ \Omega.$

**Miscellaneous Example :**

**Ex.32** Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

**Sol.** Capacitance = C




Electric field = $\frac{3Q}{2A\varepsilon_0}$

$$V = \frac{3Qd}{2A\varepsilon_0}$$

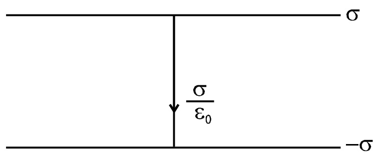
$$\Rightarrow \quad V = \frac{3Q}{2C} = \frac{\frac{3Q}{2}}{C}$$



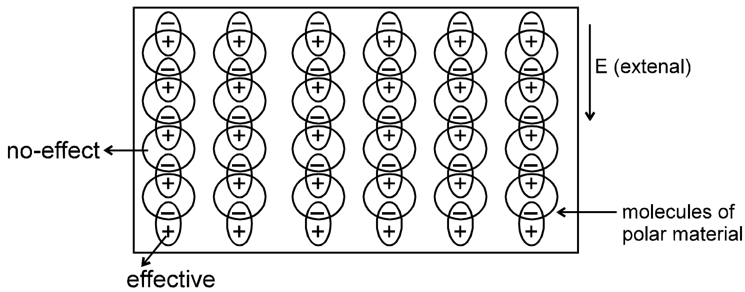
## 8. CAPACITORS WITH DIELECTRIC

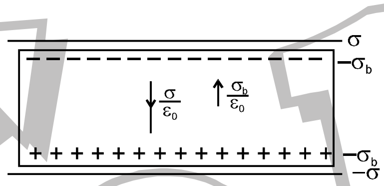
(i) In absence of dielectric



$E = \frac{\sigma}{\varepsilon_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.



$r_b$ = induced charge density (called bound charge because it is not due to free electrons).

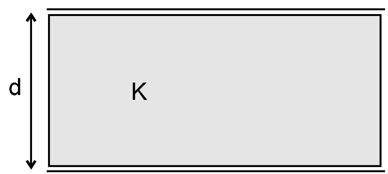
* For polar molecules dipole moment ≠ 0

* For non-polar molecules dipole moment = 0

For non-polar molecules the molecule of substance arranged as given below :

(iii) Capacitance is presence of dielectric

$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0}.d} = \frac{AK\varepsilon_0}{d} = \frac{AK\varepsilon_0}{d}$$

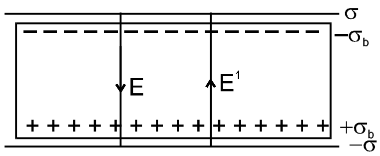



Here capacitance is increased by a factor K.

$$C = \frac{AK\varepsilon_0}{d}$$

(iv) Polarisation of material :

When nonpolar substance is placed in electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarisation of material. The induced charge also produce electric field.



$\sigma_b$ = induced (bound) charge density.

$$E_{in} = E - E_{ind} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

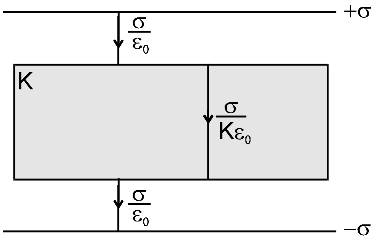
It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by $\varepsilon_r$ or k.

$$= \frac{\sigma}{K\varepsilon_0}$$

$\Rightarrow$ $$\sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

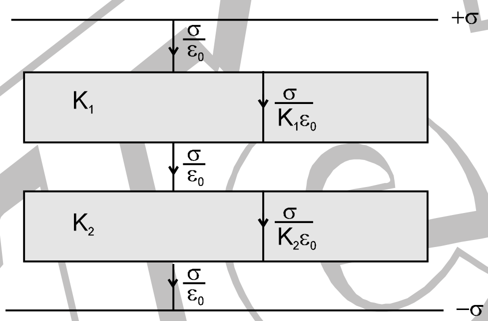
(v) If the medium does not filled between the plates completely then. Electric field will be as shown in figure

**Case : (1)**

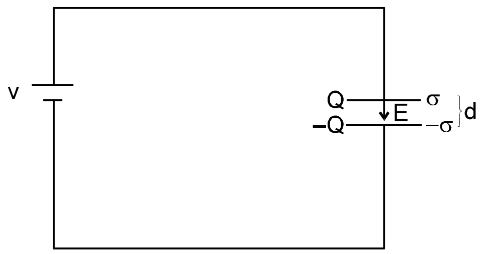



The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.

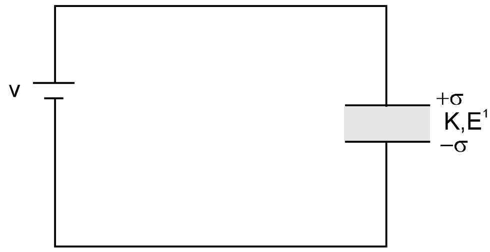
**Case : (2)**




(vi) Comparison of E (electric field), σ (surface charges density), Q (charge ), C (capacitance) and F (force between the plates) before and after inserting a dielectric slab between the plates of a parallel plate capacitor.



Case I



Case II

$C = \frac{\varepsilon_0 A}{d}$

$Q = CV$

$E = \frac{\sigma}{\varepsilon_0} = \frac{Cv}{A\varepsilon_0}$

$= \frac{V}{d}$

$C' = \frac{A\varepsilon_0 K}{d}$

$Q' = C'V$

$E' = \frac{\sigma}{K\varepsilon_0} = \frac{Cv}{A\varepsilon_0}$

$= \frac{V}{d}$ also

Here potential difference between the plates,

$Ed = V$

$E = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma}{\varepsilon_0}$

Here potential difference between the plates

$E'd$

$E' = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma'}{K\varepsilon_0}$

Equating both

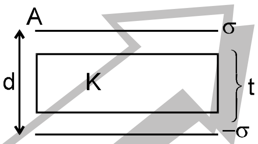
$$\frac{\sigma}{\varepsilon_0} = \frac{\sigma'}{K\varepsilon_0}$$

$$\sigma' = K\sigma$$

In the presence of dielectric, i.e. in case IInd capacitance of capacitor is more.

(vii) Energy density of a dielectric = $\frac{1}{2}\varepsilon_0\varepsilon_r E^2$

**Ex.33** If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates d (d > t) then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab?




$$C = \frac{Q}{v} = \frac{\sigma A}{v}$$

$$V = \frac{\sigma t_1}{\varepsilon_0} + \frac{\sigma t}{K\varepsilon_0} + \frac{\sigma t_2}{\varepsilon_0} \qquad (\because t_1 + t_2 = d - t)$$

$$= \frac{\sigma}{\varepsilon_0}\left[t_1 + t_2 + \frac{t}{k}\right]$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{Q}{C} = \frac{\sigma A}{C}$$

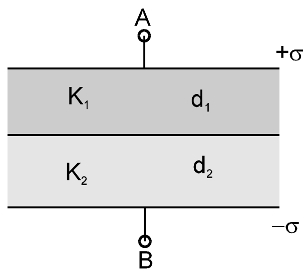
$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t + t/K}$$

**Note** **(i)** Capacitance does not depend upon the position of dielectric (it can be shifted up or down still capacitance does not change).

(ii) If the slab is of metal slab then

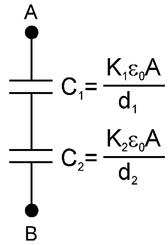
$$C - \frac{A\varepsilon_0}{d-t}$$

**Ex.34** Find out capacitance between A and B if two dielectric slabs of dielectric constant $K_1$ and $K_2$ of thickness $d_1$ and $d_2$ and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



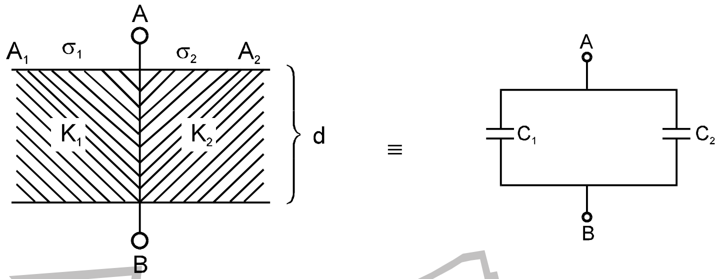
**Sol.** $C = \dfrac{\sigma A}{V}$ ; $V = E_1 d_1 + E_2 d_2 = \dfrac{\sigma d_1}{K_1\varepsilon_0} + \dfrac{\sigma d_2}{K_2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}\left(\dfrac{d_1}{k_1} + \dfrac{d_2}{k_2}\right)$

$\therefore \quad C = \dfrac{A\varepsilon_0}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}} \quad \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{AK_1\varepsilon_0} + \dfrac{d_2}{AK_2\varepsilon_0}$




This formula suggests that the system between A and B can be considered as series combination of two capacitors.

**Ex.35** Find out capacitance between A and B if two dielectric slabs of dielectric constant $K_1$ and $K_2$ of area $A_1$ and $A_2$ and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.

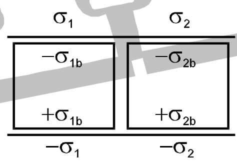



$C_1 = \dfrac{A_1K_1\varepsilon_0}{d}$, $C_2 = \dfrac{A_2K_2\varepsilon_0}{d}$

$E_1 = \dfrac{V}{d} = \dfrac{\sigma_1}{K_1\varepsilon_0}$, $E_2 = \dfrac{V}{d} = \dfrac{\sigma_2}{K_2\varepsilon_0}$

$\sigma_1 = \dfrac{K_1\varepsilon_0 V}{d} \qquad \sigma_2 = \dfrac{K_2\varepsilon_0 V}{d}$

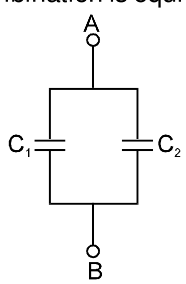
$\dfrac{\sigma_1}{K_1} = \dfrac{\sigma_2}{K_2}$




$\sigma_1 b = \sigma_1(1 - \dfrac{1}{k_1})$

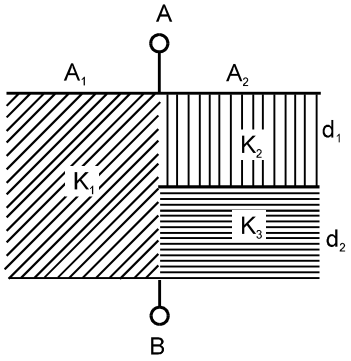
$\sigma_2 b = \sigma_2(1 - \dfrac{1}{k_2})$

The combination is equivalent to :

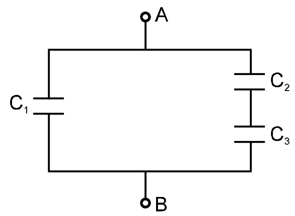



$\therefore C_1 + C_2$

**Ex.36** Find out capacitance between A and B if three dielectric slabs of dielectric constant $K_1$ of area $A_1$ and thickness d, $K_2$ of area $A_2$ and thickness $d_1$ and $K_3$ of area $A_2$ and thickness $d_2$ are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)




**Sol.** It is equivalent to



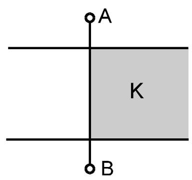

$$C = C_1 + \frac{C_2C_3}{C_2 + C_3}$$

$$C = \frac{A_1K_2\varepsilon_0}{d_1 + d_2} + \frac{\frac{A_2K_2\varepsilon_0}{d_1} \cdot \frac{A_2K_3\varepsilon_0}{d_1}}{\frac{A_2K_2\varepsilon_0}{d_1} + \frac{A_2K_3\varepsilon_0}{d_2}}$$

$$= \frac{A_1K_1\varepsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3\varepsilon_0}{A_2K_2\varepsilon_0d_2 + A_2K_3\varepsilon_0d_1}$$

$$= \frac{A_1K_2\varepsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3\varepsilon_0}{K_2d_2 + K_3d_1}$$

**Ex.37** A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-




(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$ (C) $\frac{C}{2}[1 + K]$ (D) $\frac{2[1+K]}{C}$

**Sol.** This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2$$

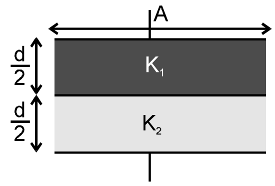
$$= \frac{\varepsilon_0 A/2}{d} + \frac{\varepsilon_0 (A/2)K}{2}$$

$$= \frac{\varepsilon_0 A}{d}\,[1 + K]$$

$$= \frac{C}{2}\,[1 + K]$$

Hence the correct answer will be (C).

**Ex.38** In the adjoining figure two dielectrics of constants $K_1$ and $K_2$ are filled in a parallel plate condenser. The capacity of the condenser will be :




(A) $C = \dfrac{\varepsilon_0 A}{d}$ (B) $C = \dfrac{2K_1K_2\varepsilon_0 A}{d(K_1+K_2)}$

(C) $C = \dfrac{2(K_1+K_2)}{2K_1K_2\varepsilon_0 A}$ (D) $\dfrac{2K_1K_2}{K_1+K_2}$

**Sol.** Two capacitors, each of area A and plate separation $\dfrac{d}{2}$, are connected in series.

$\therefore \quad C = \dfrac{C_1C_2}{C_1+C_2}$

$C_1 = \dfrac{2K_1\varepsilon_0 A}{d}$

$C_2 = \dfrac{2K_2\varepsilon_0 A}{d}$

$C = \dfrac{2K_1K_2\varepsilon_0 A}{d}$

$C = \dfrac{2K_1K_2\varepsilon_0 A}{d(K_1+K_2)}$

**Ex.39** The capacity of a parallel plate capacitor in air is 50μF and on immersing it into oil it becomes 110 μF. The dielectric constant of oil is -

(A) 0.45 (B) 0.55 (C) 1.10 (D) 2.20

**Sol.** $K = \dfrac{C}{C_0}$

$K = \dfrac{110}{50} = \dfrac{11}{5} = 2.20$

**Ex.40** Two parallel plate condensers with capacities C and 2C are connected in parallel and are charged to potential difference V. Now the battery is removed and a dielectric of constant K is inserted between the plates of condenser C. Now the potential difference across each condenser will be-

(A) $\dfrac{V}{K+2}$ (B) $\dfrac{2V}{2+K}$ (C) $\dfrac{3V}{2+K}$ (D) $\dfrac{2+K}{3V}$

**Sol.** $C' = C + 2C = 3C$

$q = C'V = 3CV$

When dielectric is inserted

$C'' = KC + 2C$

$= (K + 2)\,C$

$\therefore$ the potential difference across the capacitors

$V_f = \dfrac{\text{total charge}}{\text{total capacitance}}$

$= \dfrac{q}{C''} = \dfrac{3CV}{C(2+K)} = \dfrac{3V}{2+K}$

**Ex.41** A parallel plate condenser is charged to a certain potential and then disconnected. The separation of the plates is now increased by 2.4 mm and a plate of thickness 3 mm is inserted into it keeping its potential constant. The dielectric constant of the medium will be-

(A) 5 (B) 4 (C) 3 (D) 2

**Sol.** As charge and potential of the condenser both are constant in two cases, hence its capacity must also remain constant

$\therefore$ $C_0 = C$

or $\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left[1 - \frac{1}{K}\right]}$

or $d = d' - t\left[1 - \frac{1}{K}\right]$

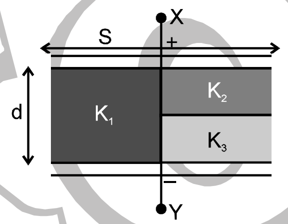
or $(d' - d) = t\left[1 - \frac{1}{K}\right]$

or $2.4 \times 10^{-3} = 3 \times 10^{-3}\left[1 - \frac{1}{K}\right]$

or $1 - \frac{1}{K} = 0.8$ or $\frac{1}{K} = 0.2$

$\therefore$ $K = 5$

**Ex.42** Three dielectric materials are filled in a parallel plate condenser as shown in figure. A potential difference is applied between the plates of the condenser. If the area of each plate is S and distance between parallel plates is d then the capacity between the point X and Y will be -




(A) $\frac{\varepsilon_0 S}{d}\left[\frac{K_1}{2} + \frac{K_2K_3}{K_2 + K_3}\right]$ (B) $\frac{\varepsilon_0 S}{d}$ (C) $\frac{\varepsilon_0 SK_1K_2K_3}{d}$ (D) $\frac{\varepsilon_0 SK_2K_3}{d}$

**Sol.** $C_1 = \frac{K_1\varepsilon_0 S/2}{d/2}$

$C_2 = \frac{K_2\varepsilon_0 S/2}{d/2}$

$C_3 = \frac{K_3\varepsilon_0 \frac{S}{2}}{\frac{d}{2}}$

$\because$ $C_2$ and $C_3$ are in series

$C' = \frac{C_2C_3}{C_2 + C_3}$

$C_1$ and $C'$ are in parallel

$C'' = C_1 + \frac{C_2C_3}{C_2 + C_3}$

$C'' = \frac{K_1\varepsilon_0 S}{2d} + \frac{K_3K_2(\varepsilon_0 S)^2/d^2}{\frac{K_2\varepsilon_0 S}{d} + \frac{K_3\varepsilon_0 S}{d}}$

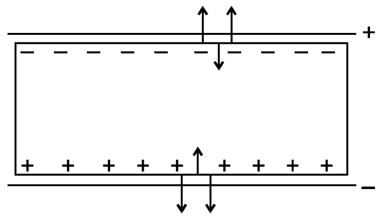


Teko Classes, Maths : Suhag R. Kariya (S. R. K. Sir), Bhopal Phone : 0 903 903 7779, 0 98930 58881. 

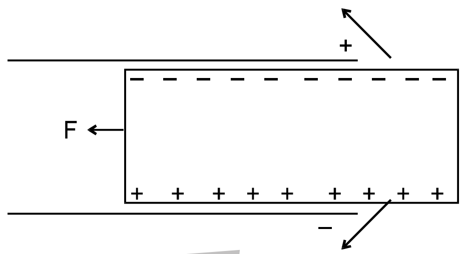
$$C'' = \frac{\varepsilon_0 S}{d}\left[\frac{K_1}{2} + \frac{K_2 K_3}{K_2 + K_3}\right]$$

Hence the correct answer will be (A).

(viii) Force on a dielectric due to charged capacitor

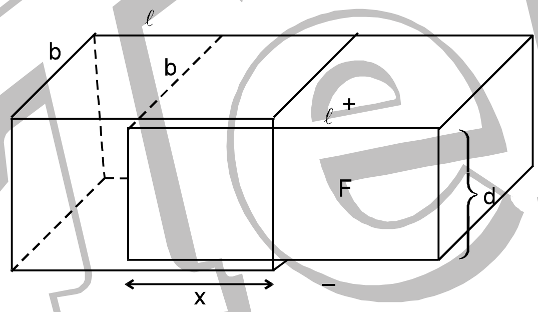



If dielectric is completely inside the capacitor then force is equal to zero.




**Case I -** Voltage source remains connected

V = constant.




$$U = \frac{1}{2} CV^2$$

$$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx}$$

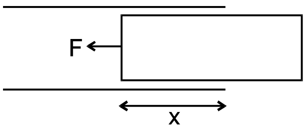
where $C = \frac{xb\varepsilon_0 K}{d} + \frac{\varepsilon_0(\ell - x)b}{d}$

$\Rightarrow$ $F = \frac{\varepsilon_0 b}{d}[Kx + \ell - x]$

$= \frac{\varepsilon_0 b}{d}(K - 1)$

$\therefore$ $F = \frac{\varepsilon_0 b(K-1)V^2}{2d}$ = constant (does not depend on x)

**Case II :** When charge on capacitor is constant




$$C = \frac{xb\varepsilon_0 K}{d} + \frac{\varepsilon_0(\ell - x)b}{d}, \qquad U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2}\cdot\frac{dC}{dx} \qquad\qquad \frac{dC}{dx} = \frac{\varepsilon_0 b}{d}(K - \ell)$$

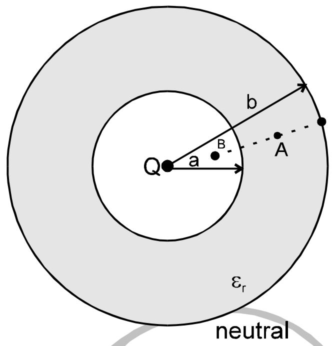
$$= \frac{Q^2}{2C^2}\cdot\frac{dC}{dx}$$

**Ex.43** Find V and E at : ( Q is a point charge kept at the centre of the nonconducting neutral thick sphere of inner radius 'a' and outer radius 'b')

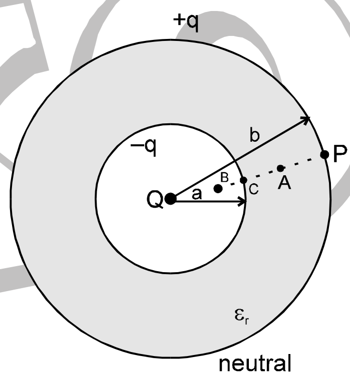
(i) $0 < r < a$

(ii) $a \le r < b$

(iii) $r \ge b$




**Sol.** –q and +q charge will induce on inner and outer surface respectively




$$E(0 < r < a) = \frac{KQ}{r^2}$$

$$E\,(r \ge b) = \frac{KQ}{r^2}$$

$$E\,(a \le r < b) = \frac{KQ}{r^2} - \frac{Kq}{r^2}$$

$$= \frac{KQ}{\varepsilon_r r^2}$$

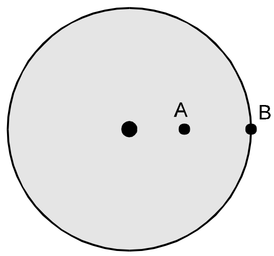
$$q = Q\left(1 - \frac{1}{\varepsilon_r}\right).$$

$$V\,(r \ge b) = \frac{KQ}{r}$$

$$(a \le r \le b)\ V_A = V_P + \int_b^r \frac{KQ}{\varepsilon_r r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\varepsilon_r}\left(\frac{1}{r} - \frac{1}{b}\right)$$

$$V\,(r \le a)\, V_B = V_C + \int_a^r \frac{KQ}{r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) + kQ\left(\frac{1}{r} - \frac{1}{a}\right)$$

**Ex.44** What is potential at a distance r (<R) in a dielectric sphere of uniform charge density $\rho$ , radius R and dielectric constant $\varepsilon_r$..




**Sol.** $V_A = V_B + \frac{W_{B\to A}}{q}$



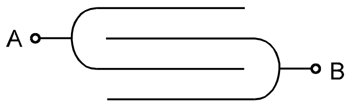
$$V = \frac{Q}{4\pi\varepsilon_0 R} + \int_R^r \frac{\rho r}{3\varepsilon_0\varepsilon_r}(-dr)$$

$$= \frac{Q}{4\pi\varepsilon_0 R} + \frac{\rho(R^2 - r^2)}{3\varepsilon_0\varepsilon_r}$$

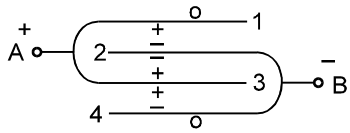
$$V_{outiside} = \frac{KQ}{r}$$

## 9. COMBINATION OF PARALLEL PLATES

**Ex.45** Find out equivalent capacitance between A and B.




**Sol.** Put numbers on the plates The charges will be as shown in the figure.

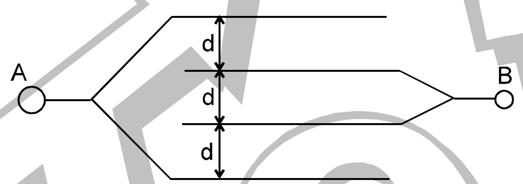



$V_{12} = V_{32} = V_{34}$

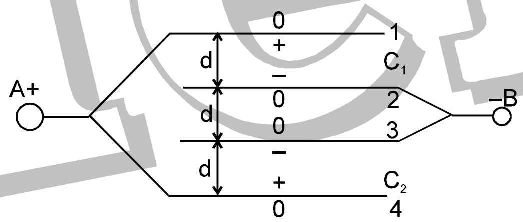
so all the capacitors are in parallel combination.

$C_{eq} = C_1 + C_2 + C_3$

**Ex.46** Find out equivalent capacitance between A and B.



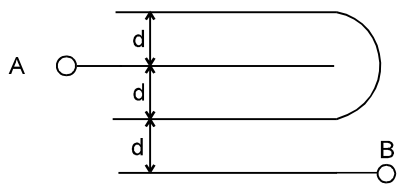

**Sol.**



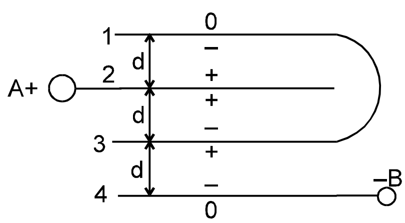

These are only two capacitors.

$C_{eq} = C_1 + C_2$

**Ex.47** Find out equivalent capacitance between A and B.



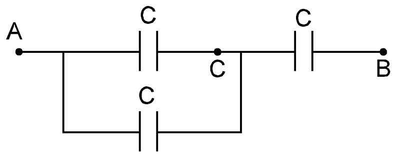

**Sol.**




The modified circuit is

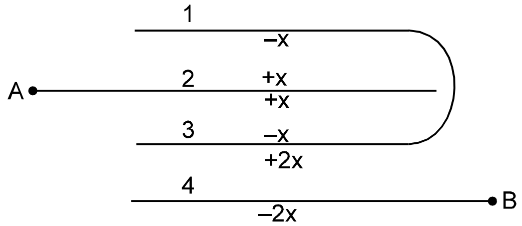





$$C_{eq} = \frac{2C}{3}$$

**Other method :**




$$C_{eq} = \frac{Q}{V}$$

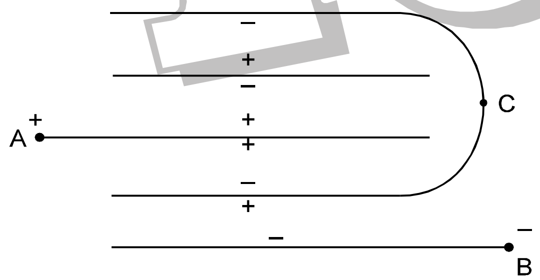
$$= \frac{2xA}{V}$$

$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_2)$$

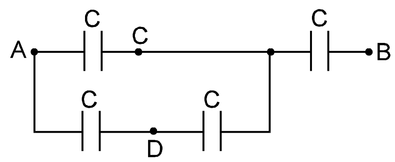
$$= \frac{xd}{\varepsilon_0} + \frac{2xd}{\varepsilon_0} = \frac{3xd}{\varepsilon_0}$$

$$\therefore \qquad C_{eq} = \frac{3xAx\varepsilon_0}{3xd} = \frac{2A\varepsilon_0}{3d} = \frac{2C}{3}.$$

**Ex.48** Find out equivalent capacitance between A and B.




**Sol.** $C = \frac{A\varepsilon_0}{d}$




$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_{eq} = \frac{3C}{5} = \frac{3A\varepsilon_0}{5d}$$

**Alternative Method :**

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$



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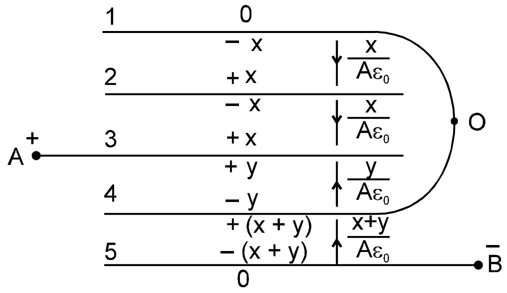



$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

Potential difference between 1 and 4 is same

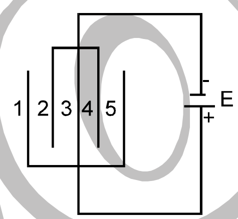
$$\frac{y}{A\varepsilon_0} = \frac{2x}{A\varepsilon_0}$$

$$y = 2x$$

$$V = \left(\frac{2y+x}{A\varepsilon_0}\right)d$$

$$C = \frac{(x+2x)xA\varepsilon_0}{(5x)d} = \frac{3A\varepsilon_0}{5d}.$$

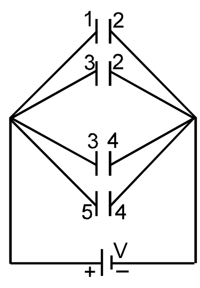
**Ex.49** Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. E as shown in the following diagram. The charge on the plates 1 and 4 will be-




(A) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 AV}{d}$

(C) $\frac{-\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ (D) $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$

**Sol.** Equivalent circuit diagram Charge on first plate




$Q = CV$

$$Q = \frac{\varepsilon_0 AV}{d}$$

Charge on fourth plate

$Q' = C(-V)$

$$Q' = \frac{-\varepsilon_0 AV}{d}$$

As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

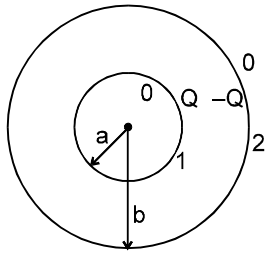
$$Q' = -\frac{2\varepsilon_0 AV}{d}$$

Hence the correct answer will be (B).

## 10. OTHER TYPES OF CAPACITORS

**Spherical capacitor :**

This arrangement is known as spherical capacitor.

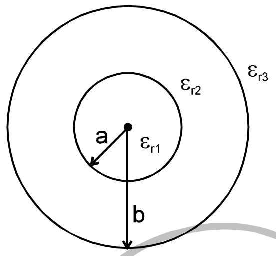



$$V_1 - V_2 = \left[\frac{KQ}{a} - \frac{KQ}{b}\right] - \left[\frac{KQ}{b} - \frac{KQ}{b}\right]$$

$$= \frac{KQ}{a} - \frac{KQ}{b}$$

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}}$$

$$= \frac{ac}{K(b-a)} \qquad = \frac{4\pi\varepsilon_0 ab}{b-a}$$



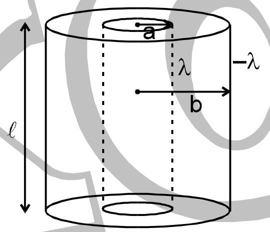

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

If b >> a

$C = 4\pi\varepsilon_0 a$

$$C = \frac{4\pi\varepsilon_0 \varepsilon_{r_2} ab}{b-a}$$

**Cylindrical capacitor**




There are two co-axial conducting cylindrical surfaces

$\ell$ >> a

>> b

where a and b is radius of cylinders.

$$C = \frac{\lambda}{V}$$

$$= \frac{\lambda}{2K\lambda \ell n\frac{a}{b}}$$

$$= \frac{4\pi\varepsilon_0}{2\ell n\frac{b}{a}}$$

$$= \frac{2\pi\varepsilon_0}{\ell n\frac{b}{a}}$$

Capacitance per unit length = $\dfrac{2\pi\varepsilon_0}{\ell n\frac{b}{a}}$ F/m

## SUMMARY



- Electric potential energy (Self Energy) :

$$U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}$$

- Energy density :

$$\frac{1}{2}\varepsilon_0 \varepsilon_r E^2$$

- Capacitance of an isolated spherical shell :

$C_{Vacuum} = 4\pi\varepsilon_0 R$

- For a parallel plate capacitor :

$$C = \frac{\varepsilon_0 A}{d}$$

$E = \sigma/\varepsilon_0 = V/d$

- In series combination :

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

- In parallel combination :

$$C_{eq} = \sum_{n=1}^{n} C_n$$

- For charging circuit :

$$q = Q_{st}\left(1 - e^{-t/\tau}\right) \; ; \; \tau = R_{eff}.\, C$$

$$i = \frac{Q_{st}}{\tau}\left(e^{-t/\tau}\right)$$

- For dicharging circuit :

$$q = Q_{max}\left(e^{-t/\tau}\right) \; ; \; \tau = R_{eff}.\, C$$

$$i = \frac{Q_{max}}{\tau}\left(e^{-t/\tau}\right)$$

- Force on a dielectric :

When battery is connected

$$F = \frac{\varepsilon_0 b(K-1)V^2}{2d}$$

- When battery is not connected

$$F = \frac{Q^2}{2C^2} . \frac{dC}{dx}$$

- Capacitance of a spherical capacitor :

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

- Capacitance of a cylinderical capacitor per unit length :

$$C = \frac{2\pi\varepsilon_0}{\ell n \frac{b}{a}}$$

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